A note on input-output linkage measures

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1st WIOD Conference, Vienna, Austria, June 25-28, 2010
1. Introduction

2. IO linkages: traditional approach

3. Generalized IO linkages

4. Empirical exercises

5. Conclusion
Applications of IO linkages (Miller and Blair, 2009, Chapter 12)
- Comparative analyses of production structures of economies
- Structural change, impact analysis
- Key sector identification (output, income, greenhouse gases, etc.)
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  - Structural change, impact analysis
  - Key sector identification (output, income, greenhouse gases, etc.)

We consider six widely used IO linkage measures
  - gross output approach
  - generalized IO setting (less used, but more relevant)
Purpose of this note

- How are the IO linkages interrelated?
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- Propose a net forward linkage as a ‘counterpart’ of Oosterhaven and Stelder’s (2002) net backward linkage measure.
Introduction

Purpose of this note

- How are the IO linkages interrelated?
- Derive the closed-form expressions for three hypothetical extraction linkages.
- Propose a net forward linkage as a ‘counterpart’ of Oosterhaven and Stelder’s (2002) net backward linkage measure.
- Empirical evidence on the link between these linkages.
Leontief model

\[ x = Ax + y, \text{ where } A = Z\hat{x}^{-1} \text{ is the input matrix. Thus,} \]

\[ x = Ly, \]  

(1)

where \( L = (I - A)^{-1} \) is the Leontief inverse (Leontief 1936, 1941).
**Leontief model**

- $x = Ax + y$, where $A = Zx^{-1}$ is the input matrix. Thus,
  
  $$x = Ly,$$

  where $L = (I - A)^{-1}$ is the Leontief inverse (Leontief 1936, 1941).

- The total backward linkage of sector $i$ is
  
  $$b_i = m_i^o = \sum_{k=1}^{n} l_{ki}.$$

Ghosh model

- \( x'B + v' = x' \), where \( B = \hat{x}^{-1}Z \) is the output matrix. Thus,

\[
x' = v'G,
\]

where \( G = (I - B)^{-1} \) is the Ghosh inverse (Ghosh 1958).
**Ghosh model**

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  x' = v' G,
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  where \( G = (I - B)^{-1} \) is the Ghosh inverse (Ghosh 1958).

- The **total forward linkage** of sector \( i \) is

  \[
  f_i = \sum_{k=1}^{n} g_{ik}.
  \]
**Hypothetical extraction method**

To find the (relative stimulative) **importance** of sector $i$, (Strassert 1968, Schultz 1977),

- delete the $i$-th row and column of the input matrix $A$,
HYPOTHETICAL EXTRACTION METHOD

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- delete the $i$-th row and column of the input matrix $A$,
- using Leontief model compute the reduced outputs (with $y_i = 0$),
Hypothetical extraction method

To find the (relative stimulative) importance of sector $i$, (Strassert 1968, Schultz 1977),

- delete the $i$-th row and column of the input matrix $A$,
- using Leontief model compute the reduced outputs (with $y_i = 0$),
- find the difference between total output before and after the extraction, i.e,
  total linkage of sector $i = \nu'x - \nu'x^{-i}$ ($\nu$ - summation vector).
Non-complete HEM

Distinguish between backward and forward HE linkages (Dietzenbacher and van der Linden 1997)
Non-complete HEM

- Distinguish between backward and forward HE linkages
  (Dietzenbacher and van der Linden 1997)
- Denote
  $A^{-i}_c$ - nullify $i$-th column of $A$
  $B^{-i}_r$ - nullify $i$-th row of $B$
Non-complete HEM

- Distinguish between backward and forward HE linkages (Dietzenbacher and van der Linden 1997)
- Denote
  \[ A_{c}^{-i} \text{ - nullify } i\text{-th column of } A \]
  \[ B_{r}^{-i} \text{ - nullify } i\text{-th row of } B \]
- Then, \[ x_{c}^{-i} = (I - A_{c}^{-i})^{-1}y \] and \[ (x_{r}^{-i})' = v'(I - B_{r}^{-i})^{-1} \]
Non-complete HEM

- Distinguish between backward and forward HE linkages (Dietzenbacher and van der Linden 1997)
- Denote
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- Then, \[ x_{c}^{-i} = (I - A_{c}^{-i})^{-1}y \] and \[ (x_{r}^{-i})' = v'(I - B_{r}^{-i})^{-1} \]
- Hypothetical extraction backward and forward linkages of \( i \) are

\[
  b_{i}^{h} = \iota'x - \iota'x_{c}^{-i} \quad \text{and} \quad f_{i}^{h} = \iota'x - \iota'x_{r}^{-i}
\]  \hspace{1cm} (5)
Net backward linkage

Oosterhaven and Stelder’s (2002) key sector indicator

- Net backward linkage of sector $i$

$$b_i^n = \frac{b_i y_i}{x_i},$$ (6)

i.e., output generated in all sectors due to final demand of sector $i$
output generated in sector $i$ due to final demands of all sectors.
What is the closed form of the complete HEM linkage, $\nu'x - \nu'x^{-i}$?

It is given by the output worth $\omega_i = b_i x_i l_{ii}$ (Temurshoev 2010).
Output worth measure

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Output worth measure

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(7)

- Besides multiplier \( b_i \), the size \( x_i \) and input self-dependency \( l_{ii} \) are considered
What is the closed form of the complete HEM linkage, $\nu'x - \nu'x^{-i}$?

It is given by the output worth of $i$ (Temurshoev 2010)

$$\omega_i^o = \frac{b_i x_i}{l_{ii}} \quad (7)$$

Besides multiplier $b_i$, the size $x_i$ and input self-dependency $l_{ii}$ are considered.

Three step HEM procedure is redundant.
Non-complete HEM linkages

What about $b_{i}^{h} = \nu'x - \nu'x_{c}^{-i}$ and $f_{i}^{h} = \nu'x - \nu'x_{r}^{-i}$?

Result 1

The closed-form expressions of the backward and forward linkages resulting from the hypothetical extraction of sector $i$ are given, respectively, by

$$b_{i}^{h} = \frac{(b_{i} - 1)x_{i}}{l_{ii}} \quad \text{and} \quad f_{i}^{h} = \frac{(f_{i} - 1)x_{i}}{l_{ii}}. \quad (8)$$

No extraction needed (e.g., PyIO). Note that $\omega_{i}^{o} = b_{i}^{h} + \frac{x_{i}}{l_{ii}}$. 
Connections between the IO linkages:

**Result 2**

The following identities between the linkage measures always hold:

\[
\begin{align*}
    b_i^h &= \frac{\omega_i(b_i - 1)}{b_i} = \frac{\omega_i^o(b_i - 1)}{b_i^n} s_i \\
    f_i^h &= \frac{\omega_i(f_i - 1)}{b_i} = \frac{\omega_i^o(f_i - 1)}{b_i^n} s_i,
\end{align*}
\]

where \( s_i = y_i/x_i \) is the share of net output in gross output of sector \( i \).
Let $\pi$ be the vector of direct factor coefficients
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- The total factor backward linkage of sector $i$ is $(b'_\pi = \pi'L)$

$$b_i^\pi = m_i^\pi = \sum_{k=1}^{n} \pi_k l_{ki}. \quad (10)$$
**Generalized IO linkages**

- Let $\pi$ be the vector of direct factor coefficients
- The total factor backward linkage of sector $i$ is $(b'_\pi = \pi' L)$

\[
b^\pi_i = m^\pi_i = \sum_{k=1}^{n} \pi_k l_{ki}.
\]

(10)

- The total factor forward linkage of sector $i$ is $(f_\pi = G\pi)$

\[
f^\pi_i = \sum_{k=1}^{n} g_{ik} \pi_k.
\]

(11)
Generalized IO linkages

- Let $\pi$ be the vector of direct factor coefficients
- The total factor backward linkage of sector $i$ is ($b_\pi' = \pi'L$)

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- The total factor forward linkage of sector $i$ is ($f_\pi = G\pi$)

$$f_i^\pi = \sum_{k=1}^{n} g_{ik} \pi_k. \quad (11)$$

- Note: total factor requirements to satisfy $y = \text{total factor usage}$ accompanying $v$, i.e.,

$$b_\pi'y = \pi'x = x'\pi = v'f_\pi.$$
Oosterhaven and Stelder’s factor net backward linkage of \( i \) is

\[
b_{i}^{\pi,n} = \frac{b_{i}^{\pi} y_{i}}{\pi_{i} x_{i}},
\]

i.e., factor generated in all sectors due to final demand of sector \( i \)
factor generated in sector \( i \) due to final demands of all sectors.
Oosterhaven and Stelder’s factor net backward linkage of $i$ is

$$b_i^{\pi,n} = \frac{b_i^{\pi}y_i}{\pi_iX_i},$$

(12)

i.e., factor generated in all sectors due to final demand of sector $i$

factor generated in sector $i$ due to final demands of all sectors.

What is the closed-form of the ‘generalized’ HEM indicator, $\pi'x - \pi'x^{-i}$?
Oosterhaven and Stelder’s factor net backward linkage of $i$ is

$$b_{i}^{\pi,n} = \frac{b_{i}^{\pi}y_{i}}{\pi_{i}x_{i}},$$  \hspace{2cm} (12)

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factor generated in sector $i$ due to final demands of all sectors.

What is the closed-form of the ‘generalized’ HEM indicator, $\pi'x - \pi'x^{-i}$?

It is given by the factor worth of $i$ (Temurshoev 2010)

$$\omega_{i}^{\pi} = \frac{b_{i}^{\pi}x_{i}}{l_{ii}}.$$  \hspace{2cm} (13)
What about the ‘generalized’ non-complete HEM linkages, 

\[ b^\pi,h_i = \pi'x - \pi'x^{-i}_c \quad \text{and} \quad f^\pi,h_i = \pi'x - \pi'x^{-i}_r \tag{14} \]

**Result 3**

The closed-form expressions of the hypothetical extraction factor backward and forward linkages of sector \( i \) are given, respectively, by

\[ b^\pi,h_i = \frac{(b^\pi_i - \pi_i)x_i}{l_{ii}} \quad \text{and} \quad f^\pi,h_i = \frac{(f^\pi_i - \pi_i)x_i}{l_{ii}}. \tag{15} \]

Thus, \( \omega^\pi_i = b^\pi,h_i + \frac{\pi_i x_i}{l_{ii}}. \)
Interconnections between the linkage measures $b_i^\pi$, $f_i^\pi$, $b_i^{\pi,n}$, $\omega_i^\pi$, $b_i^{\pi,h}$ and $f_i^{\pi,h}$:

**Result 4**

The following identities between the linkage measures always hold:

$$
\begin{align*}
    b_i^{\pi,h} &= \frac{\omega_i^\pi (b_i^\pi - \pi_i)}{b_i^\pi} = \frac{\omega_i^\pi}{b_i^{\pi,n}} \frac{b_i^\pi - \pi_i}{\pi_i} s_i, \\
    f_i^{\pi,h} &= \frac{\omega_i^\pi (f_i^\pi - \pi_i)}{b_i^\pi} = \frac{\omega_i^\pi}{b_i^{\pi,n}} \frac{f_i^\pi - \pi_i}{\pi_i} s_i,
\end{align*}
$$

(16)

where $s_i = y_i/x_i$ is the share of net output in gross output of sector $i$. 
Forward ‘counterpart’ of Oosterhaven and Stelder’s net backward linkage:

- The factor net forward linkage of sector $i$ is

$$f_i^\pi, n = \frac{V_i f_i^\pi}{\pi_i X_i},$$

i.e., factor usage by all sectors associated with value-added of sector $i$ divided by factor used by sector $i$ accompanying value-added of all sectors.
Forward ‘counterpart’ of Oosterhaven and Stelder’s net backward linkage:

- The factor net forward linkage of sector $i$ is

$$f_{i}^{\pi,n} = \frac{v_{i} f_{i}^{\pi}}{\pi_{i} x_{i}},$$

i.e., factor usage by all sectors associated with value-added of sector $i$ factor used by sector $i$ accompanying value-added of all sectors.

- Hence, with $\pi_{k} = 1$ for all $k$, net forward linkage of sector $i$ is

$$f_{i}^{n} = \frac{v_{i} f_{i}}{x_{i}}.$$
Interconnections between the linkage measures involving $f_{i}^{\pi,n}$:

**Result 5**

The following identities between the linkage measures always hold:

$$b_{i}^{\pi,h} = \frac{\omega_{i}^{\pi}}{f_{i}^{\pi,n}} \frac{f_{i}^{\pi} b_{i}^{\pi} - \pi_{i}}{b_{i}^{\pi}} \tilde{s}_{i}, \quad \text{and} \quad f_{i}^{\pi,h} = \frac{\omega_{i}^{\pi}}{f_{i}^{\pi,n}} \frac{f_{i}^{\pi} f_{i}^{\pi} - \pi_{i}}{b_{i}^{\pi}} \tilde{s}_{i},$$  \hspace{1cm} (18)

where $\tilde{s}_{i} = v_{i}/x_{i}$ is the share of sector $i$’s value-added in its gross output.
Empirical exercises

**Empirical exercises**

- How similar (or different) are the outcomes of the linkage indicators?
- *The OECD IO Database*, 2006 edition: 48 sectors (commodities), two time periods
- Six countries: Austria, Greece, India, Indonesia, Japan, and the USA
- Compute output and income linkages
- Compute Spearman’s rank correlation coefficients
### Empirical exercises

#### Rank correlations of gross output IO linkages

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Conclusions

1. We discuss how IO linkages are interrelated
2. Provide closed-form expressions for three HEM linkage measures
3. Propose a factor net forward linkage
4. Some are highly correlated: (1) $\omega_i^\pi$, $b_i^\pi$, and $f_i^\pi$; (2) $\omega_i^\pi$ and $b_i^\pi,n$; (3) $f_i^\pi$ and $f_i^\pi,n$; (4) $b_i^\pi,n$, on average, is negatively correlated with the class of forward linkages
5. Each linkage is intended to address different goal (stemming from their economic interpretations), thus as expected in general there is no consistent relations between these indicators.
Thanks for your attention!